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Sparse identification of linear parameter-varying systems using B-splines

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1 Introduction

Linear parameter-varying (LPV) systems are nonlinear systems described by a linear model with coefficients varying as a function of one or more scheduling parameters. Determining the dependency of the linear model on the scheduling parameter(s), generally described through a set of basis functions, is often challenging, especially if little or no prior information of the nonlinear system dynamics is available. This abstract proposes an LPV identification method that models the scheduling parameter dependency through B-spline basis functions. B-splines are known to have better interpolating properties than polynomials. Furthermore, B-spline-based LPV models are the core of the recently developed LPV controller design method [1] we want to comply with. This LPV identification approach combines a quadratic fitting criterion with a basis pursuit approach to determine the optimal B-spline knot locations.

2 B-spline-based state-space model

We consider the following discrete-time LPV model:

$$\begin{cases} x(t+1) = A(\alpha(t))x(t) + B(\alpha(t))u(t) \\ y(t) = C(\alpha(t))x(t) + D(\alpha(t))u(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$, $y(t) \in \mathbb{R}^l$, and $\alpha(t) \in \mathbb{R}$, are respectively the state vector, the input vector, the output vector, and the scheduling parameter, at time instance t. For simplicity is here assumed that α is a scalar on a closed and bounded interval $[\underline{\alpha}, \overline{\alpha}] \subset \mathbb{R}$. The extension to multiple scheduling parameters is trivial though.
Let $\xi = (\xi_0, ..., \xi_{l+1})$ be a sequence of points satisfying

$$\underline{\alpha} = \xi_0 < \xi_1 < ... < \xi_l < \xi_{l+1} = \overline{\alpha}. \tag{2}$$

The state-space matrices of the model (1) have a piecewise polynomial dependency on α and can all be expressed as:

$$S(\alpha) = \sum_{i=1}^{n_\lambda - g - 1} C_i B_{i,g,\lambda}(\alpha), \tag{3}$$

where $B_{i,g,\lambda}(\alpha)$ is the i^{th} normalized B-spline basis function of degree g for the knot sequence $\lambda \in \mathbb{R}^{n_\lambda}$

$$(\underbrace{\xi_0, ..., \xi_0}_{g+1}, \underbrace{\xi_1, ..., \xi_1}_{g+1-v}, ..., \underbrace{\xi_l, ..., \xi_l}_{g+1-v}, \underbrace{\xi_{l+1}, ..., \xi_{l+1}}_{g+1}), \tag{4}$$

and C_i, $i = 1, ..., n_\lambda - g - 1$ are matrix-valued coefficients [1]. The B-splines $B_{i,g,\lambda}(\alpha)$, $\alpha \subset [\underline{\alpha}, \overline{\alpha}]$, are computed using the Cox-de Boor recursive formula [2].

3 Identification strategy

The procedure we propose starts with obtaining an initial model estimate using the SMILE technique [3], with B-spline basis functions of a desired degree and the knots placed at the scheduling points of the local measurements. After the initial estimate is determined, additional knots are inserted into the knot sequence, which elevates the model's degree of freedom without changing the shape of its splines. With this knot elevation, we augment the fitting capabilities of the model, aiming to obtain an as simple as possible scheduling parameter dependency while preserving or improving the LPV model accuracy. This dual goal is pursued by combining the classical weighted squared difference between the model response and corresponding measured system response focusing on the model accuracy, with a weighted $\ell_{2,1}$-norm regularization in charge of removing redundant knots.

For a polynomial spline matrix $S(\alpha)$ of degree g with internal break points $\xi_1, ..., \xi_l$ and continuity condition v it holds that in a breakpoint ξ_i, $i \in \{1, ..., l\}$, $S(\alpha)$ and its derivatives up to the order v are continuous [1]. The differentiation between the significant and redundant knots is based on the differences between adjacent elements of the v^{th} derivative and it involves specific grouping of the model coefficients followed by the $\ell_{2,1}$-norm regularization.

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